

1.2 PERMUTATIONS

You will be required to determine the number of permutations of n elements taken r at a time.

Lesson 1.2.1 (Click Here)

- A **permutation** of a set of objects is a *unique* ordering of the set. If you cannot differentiate between two orderings of the elements in a set, then you are looking at two representations of a single permutation.

For example: CAT and TCA represent two different permutations of the letters in the set {C, A, T}, but ADD and ADD represent a single permutation of the letters in the set {D, A, D} regardless of whether or not the D's changed position. This is because they cannot be differentiated.

- The number of permutations of a set with n elements can be determined using the Fundamental Counting Principle.

Illustration

Suppose we have a family of 4, consisting of two parents and two children. If we are interested in determining the number of different ways they can stand in order for a photo, then we are interested in the number of permutations of the family.

This can be calculated using the Fundamental Counting Principle. First consider the number and nature of the subtasks. We will need to find a person to stand in each of the four designated positions. Since we cannot repeat individuals, the number of options we have at each step can be represented by the expression...

$$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$$

We may conclude from this that the number of permutations of n objects is the product of all positive integers from n down to 1. This represents the number of options we have for each position. So we might say that the number of permutations of n objects is...

$$n(n-1)(n-2)(n-3) \times \dots \times 3 \times 2 \times 1$$

Since this pattern of multiplication occurs so frequently we use the ! symbol to indicate when it is occurring, and we refer to this as a factorial. Thus we will write

$$n! = n(n-1)(n-2)(n-3) \times \dots \times 3 \times 2 \times 1$$

and call it n factorial.



Watch the video **Appropriately Placing Factorials**

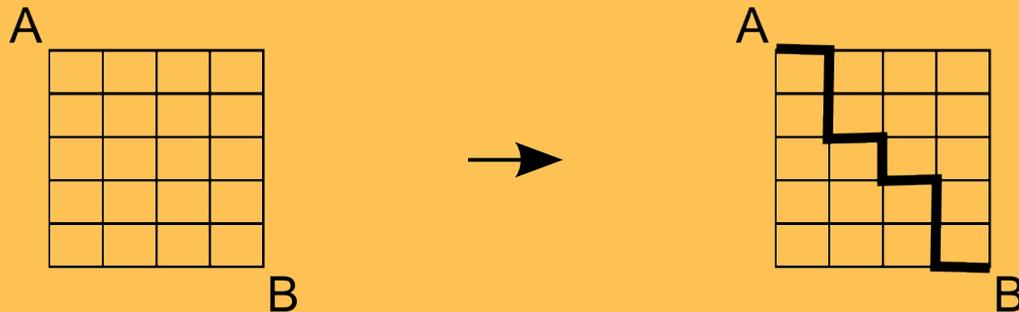


Lesson 1.3.2 (Click Here)

- One application of combinations is in the solving of pathway problems. In this course specifically the problems will focus on simple grids and combinations of grids. The rules are very straight forward.
 - You are only allowed to move toward the goal.
 - The path is determined by the line segments you cross, not the nodes you pass through. This means that your pathway will always consist of a series of vertical and horizontal line segments along which you are only allowed to travel one direction.

Illustration

In the following grid we are trying to find the number of pathways from point A to point B. And in the second illustration a specific pathway is represented.



Because the rule is that every path must always move in the direction of the goal, every possible path from A to B must cover exactly 9 line segments, 4 of which are eastward, and 5 of which are southward. The pathway represented above can be represented as...

E, S, S, E, S, E, S, S, E

The above representation opens up the possibility of two different approaches to this problem.

1. We could treat this pathway problem as a problem permuting a word with 9 letters, 4 of which are E's and 5 of which are S's. Our solution then looks like this:

$$\frac{9!}{4!5!}$$

2. Another way of looking at this uses combinations. There are 9 movements required to arrive at our goal, but there are only two options for each movement. We could either think of this as a problem where we choose 4 of the nine movements to be E, or we can choose 5 of the movements to be S. Regardless of how we look at it, because there are only the two options, the result is the same. Thus we could write:

$${}_9C_4 \text{ or } {}_9C_5$$

Notice that ${}_9C_4 = {}_9C_5 = \frac{9!}{4!5!}$.

- In a grid whose area would be calculated as $m \times n$, the number of pathways from one corner to the diagonally opposite corner can be calculated as:

$${}_{m+n}C_m \text{ or } {}_{m+n}C_n \text{ or } \frac{(m+n)!}{m!n!}$$